OPTIMUM AREA ALLOCATION FOR RESISTORS AND CAPACITORS IN CONTINUOUS-TIME MONOLITHIC FILTERS

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ABSTRACT

In this paper, the issue of optimal area allocation for passive components in monolithic active-RC filters is formalized. Optimal area allocations for several popular first- and second-order structures are derived. A conjecture is presented for optimal area allocation for more general filter structures.

1. INTRODUCTION

As part of a typical interface between the analog world and the digital world, continuous-time filters are widely used for antialiasing and reconstruction. Considerable demand exists for more general integrated filters as well. The most popular technique for building audio frequency continuous-time filters is to use either a Gm-C or a MOSFET-C approach. There are two main limitations of these types of filters. The first limitation is the process and temperature dependence of both the transconductance elements and the capacitors. This limitation is often overcome by electronically tuning or adapting either the transconductance elements or the capacitors and several techniques that give reasonable performance have been reported in the literature [1]. The other limitation, poor linearity, has been a problem for many years and although some linearization schemes have been proposed, nonlinearity remains a serious limitation of these two types of filters. These limitations were the major reasons that the switched-capacitor technique has evolved. Specifically, the switched capacitor circuits are known to have very precisely controlled pole and zero frequencies and good linearity. Switched capacitor filters are not without limitations. Switched capacitor filters inherently provide switching noise, require accurate clock generation, and are inherently discrete time rather than continuous-time in nature thus presenting aliasing of high-frequency noise into the frequency band of interest. In some semiconductor processes, good capacitors and/or good switches may not be available as well.

The conventional active RC filters are inherently linear and operate in the continuous-time domain and both of these properties are particularly attractive for a host of applications [2]. There are, however, two problems with audio frequency monolithic active RC filters. The first problem is the control of the RC products. In some anti-aliasing applications this may not be of concern and the tuning techniques that are used to tune gm-C and MOSFET-C filters can be readily adapted to tune active RC filters. The second problem is the area required to implement the passive components. Specifically, the large RC time constants needed to operate at audio frequencies range invariably require large valued resistors or large valued capacitors or both. A new strategy has been recently introduced to substantially reduce the total resistance or capacitance values needed to realize a given RC product [3]. Then, the issue of minimizing the area required to realize the passive elements becomes increasingly important if these filters are to become practical in the audio frequency applications.

Closed-form solutions have shown that for some simple first-order filter structures, the active area will be minimized if the resistor area equals the capacitor area. Conventional wisdom also suggests that this may be true in a more general case as well but there does not appear to be any formal mathematical prove to support this wisdom in literature and often this wisdom has been gathered from computer simulations for specific filter structures. In the previous works, there were no considerations for optimizing the total area for passive elements in literatures even when area is not an ignorable issue, whether for high frequency range RC filters [4] or lower frequency applications [5]. In the following section, the optimal area allocation for several different first-order and second-order active filters is investigated.

2. OPTIMUM PASSIVE AREA OF THE FIRST ORDER STRUCTURE

2.1 Single Pole Circuit

Single pole system will be discussed in this section.

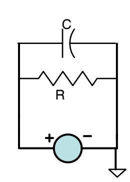

Figure 1 Single Pole Circuit

The total passive area is the summation of the area of the resistor R and the capacitor C. If the resistance density and capacitance density are R_d and C_d respectively, the total area $A_T = A_R + A_C$ is given by the expression

$$A_T = \frac{R}{R_d} + \frac{C}{C_d} \tag{1}$$

Since the pole is determined by the RC product, we introduce the constraint RC=K where K is a predetermined value. The optimum area allocation is obtained by taking the partial derivatives of A_T with respect to R and C and then setting these equal to zero subject to the constraint mentioned. We obtain the well-known result for minimizing the total active area $A_R = A_C$

This equal area allocation is independent of the value for K, R_d and C_d.

2.2 First Order Active Filter

We will next consider the first-order active filter comprising two resistors and one capacitor shown in Fig. 2.

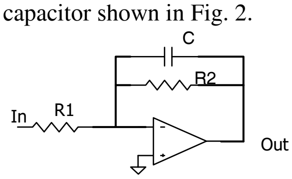

Figure 2 First Order Filter

The transfer function of this filter is given by the expression

$$\frac{V_o}{V_i} = -\frac{\frac{1}{R_1 C}}{s + \frac{1}{R_2 C}} \quad (2)$$

For this transfer function, there are three design variables, R_1, R_2, and C. If the bandwidth and DC gain are set, we will have two constrains:

$$\frac{R_2}{R_1} = K_1 \quad (3)$$

$$R_2 C = K_2 \quad (4)$$

where, K_1 and K_2 are constants. It follows that the total area for the RC constants is given by the expression

$$A_T = A_R + A_c = \frac{R_1 + R_2}{R_d} + \frac{C}{C_d} \quad (5)$$

where, A_T is the total area, A_R is the total area for the resistors and A_c is the total area for capacitors. Similarly, R_d and C_d are resistor density and capacitance density, respectively.

Using Lagrangian multipliers, to embed the constraints, we build the Hamiltonian function as shown below:

$$H = A_T + \lambda_1\left(\frac{R_2}{R_1} - K_1\right) + \lambda_2(R_2 C - K_2) \quad (6)$$

Taking the partial derivatives with respect to R_1 and R_2 we obtain the expressions

$$\frac{\partial H}{\partial R_1} = \frac{1}{R_d} + \lambda_1\left(-\frac{R_2}{R_1^2}\right) \quad (7)$$

$$\frac{\partial H}{\partial R_2} = \frac{1}{R_d} + \lambda_1\left(\frac{1}{R_1}\right) + \lambda_2 C \quad (8)$$

Setting these two derivatives to zero we obtain

$$\frac{R_1}{R_d} = \frac{\lambda_1 R_2}{R_1} = \lambda_1 K_1 \quad (9)$$

$$\frac{R_2}{R_d} = \frac{-\lambda_1 R_2}{R_1} - \lambda_2 C R_2 = -\lambda_1 K_1 - \lambda_2 K_2 \quad (10)$$

It thus follows that the total resistor area is given by

$$A_R = A_{R_1} + A_{R_2} = \frac{R_1 + R_2}{R_d} = \lambda_1 K_1 - \lambda_1 K_1 - \lambda_2 K_2 = -\lambda_2 K_2 \quad (11)$$

The same operations can be applied to the capacitor resulting in

$$\frac{\partial H}{\partial C} = \frac{1}{Cd} + \lambda_2 R_2 = 0 \quad (12)$$

Thus, we obtain the total capacitor area

$$A_C = \frac{C}{C_d} = -\lambda_2 K_2 \quad (13)$$

Comparing (11) and (13), it shows that the minimal area allocation is achieved when $A_C = A_R$. Note that this optimal area allocation is again independent of constrains K_1 and K_2.

2.3 First Order Filter with Transconductance T-network

The previous filter structure requires a big component ratio to achieve large dc gains. This big component ratio invariably requires a large area to realize the large resistor. Since the feedback resistor in the previous filter simply serves as a "transimpedance" element, the issue of maintaining the same overall transfer function with lower component ratios using alternative transimpedance elements deserves attention. A modification of this first order filter using a T-network to replace the resistor will reduce the area dramatically while keeping very good linearity [3]. The modified networks are shown in Fig.3.and Figure4.

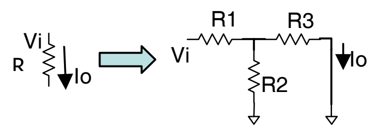

Figure 3 T-network

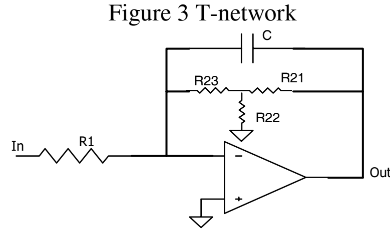

Figure 4 First Order Filter with T-network

The relationship between the resistor R_2 and the corresponding resistors of the T-network is given by the expression

$$R_2 = \frac{R_{21}}{R_{22}} R_{23} + R_{21} + R_{23} \quad (14)$$

If R_{22} is sufficiently small, equation (14) can be reduces to the approximate relationship

$$R_2 \cong \frac{R_{21}}{R_{22}} R_{23} \quad (15)$$

For the appropriate values of R_{21}, R_{22} and R_{23}, a dramatic reduction in the area required to realize the overall "transimpedance" R_2 can be achieved.

To find the optimum area allocation for this circuit, observe there are now four resistor values and one capacitor values with only two constraints, dc gain and pole frequency. We will now establish the 4 constraints by setting the ratio of R_{21} over R_{22} as K_3 and R_{21} over R_{23} as K_4

$$\frac{K_3 R_{23} + R_{23} + R_{21}}{R_1} = K_1 \quad (16)$$

$$(K_3 R_{23} + R_{21} + R_{23})C = K_2 \quad (17)$$

$$\frac{R_{21}}{R_{22}} = K_3 \quad (18)$$

$$\frac{R_{21}}{R_{23}} = K_4 \tag{19}$$

building the Hamiltonian function as below:

$$\begin{aligned} H = A_T &+ \lambda_1\left(\frac{K_3R_{23} + R_{21} + R_{23}}{R_1} - K_1\right) \\ &+ \lambda_2[(K_3R_{23} + R_{21} + R_{23})C - K_2] \\ &+ \lambda_3\left(\frac{R_{21}}{R_{22}} - K_3\right) + \lambda_4\left(\frac{R_{21}}{R_{23}} - K_4\right) \end{aligned} \tag{20}$$

Taking the partial derivatives of R_1, R_{21}, R_{22}, R_{23}, and C and set these equations to zero, we have

$$\frac{\partial H}{\partial R_1} = \frac{1}{R_d} + \lambda_1\left(-\frac{K_3R_{23} + R_{21} + R_{23}}{R_1^2}\right) = 0 \tag{21}$$

$$\Rightarrow \frac{R_1}{R_d} = \lambda_1 \frac{K_3R_{23} + R_{21} + R_{23}}{R_1} = \lambda_1 K_1 \tag{22}$$

$$\frac{\partial H}{\partial R_{21}} = \frac{1}{R_d} + \lambda_1\left(\frac{1}{R_1}\right) + \lambda_2(C) + \lambda_3\left(\frac{1}{R_{22}}\right) + \lambda_4 \frac{1}{R_{23}} = 0 \tag{23}$$

$$\Rightarrow \frac{R_{21}}{R_d} = -\lambda_1 \frac{R_{21}}{R_1} - \lambda_2 R_{21} C - \lambda_3 K_3 - \lambda_4 K_4 \tag{24}$$

$$\frac{\partial H}{\partial R_{22}} = \frac{1}{R_d} - \lambda_3\left(\frac{R_{21}}{R^2{}_{22}}\right) = 0 \tag{25}$$

$$\Rightarrow \frac{R_{22}}{R_d} = \lambda_3 \frac{R_{21}}{R_{22}} = \lambda_3 K_3 \tag{26}$$

$$\frac{\partial H}{\partial R_{23}} = \frac{1}{R_d} + \lambda_1\left(\frac{1+K_3}{R_1}\right) + \lambda_2(1+K_3)C - \lambda_4 \frac{R_{21}}{R_{23}{}^2} = 0 \tag{27}$$

$$\Rightarrow \frac{R_{23}}{R_d} = -\lambda_1 \frac{(1+K_3)R_{23}}{R_1} - \lambda_2(1+K_3)R_{23}C + \lambda_4 K_4 \tag{28}$$

from constrains (16) and (17), we can derive:

$$(1+K_3)R_{23} = K_1R_1 - R_{21} \tag{29}$$

$$(1+K_3)R_{23} = \frac{K_2}{C} - R_{21} \tag{30}$$

replacing (29) and (30) into the equation (28), we have

$$\Rightarrow \frac{R_{23}}{R_d} = -\lambda_1 K_1 + \lambda_1 \frac{R_{21}}{R_1} - \lambda_2 K_2 + \lambda_2 R_{21} C + \lambda_4 K_4 \tag{31}$$

$$A_R = (22) + (24) + (26) + (31) = -\lambda_2 K_2 \tag{32}$$

$$\frac{\partial H}{\partial C} = \frac{1}{C_d} + \lambda_2(K_3R_{23} + R_{21} + R_{23}) = 0 \tag{33}$$

$$A_C = \frac{C}{C_d} = -\lambda_2(K_3R_{23} + R_{21} + R_{23})C = -\lambda_2 K_2 \tag{34}$$

It clearly shows that the conjecture presented above is true for the first-order filter with the T-network and it will reduce the passive area further. Now the question is if that is the truth for alternate structure for higher order filters? The following part is focused on the several most popular second-order filters.

3. MINIMIZING THE PASSIVE AREA FOR SECOND-ORDER FILTERS

Three second-order filters will now be presented in this section. These are the Tow-Thomas Biquad, the Tow-Thomas Biquad with a T network to reduce component spread, and the bridged-T feedback structure. The Tow-Thomas biquad and the bridge-T feedback structures have been widely used for implementing analog filters. The modified Tow-Thomas biquad is used for building high-linear integrated audio frequency filters within fairly small area [2]. These structures were chosen because they use different numbers of passive components, have different component rations, and use different numbers of op amps.

3.1 Tow-Thomas Biquad

The first example is Tow-Thomas biquad.

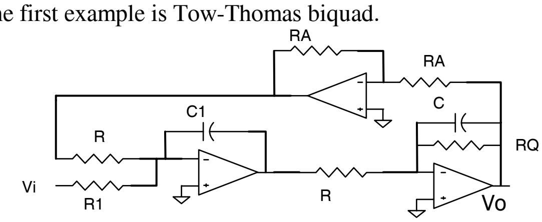

Figure 5 Tow-Thomas Biquad

The transfer function is shown as below:

$$\frac{V_o}{V_i} = \frac{\frac{R}{R_1}\left(\frac{1}{R^2CC_1}\right)}{S^2 + \frac{S}{R_QC} + \frac{1}{R^2CC_1}} \tag{35}$$

$$A_T = A_R + A_C = \frac{2R + R_1 + R_Q}{R_d} + \frac{C_1 + C}{C_d} \tag{36}$$

In this implementation, we have assumed the integration resistors are equal. We will not consider any resistor area associated with the resistors used to realize the finite gain amplifier, that is, the area required to realize the two R_A resistors will be ignored. We will ignore these resistors because we view this as a dimensionless gain block. Thus, we will consider the area related with the components that determine the poles of the network, specifically the two R resistors, the R_Q resistor and the capacitors C and C_1.

If the w_0, Q and dc gain are all set, we have three constrains:

$$R_QC = K_1 \tag{37}$$

$$R^2CC_1 = K_2 \tag{38}$$

$$\frac{R}{R_1} = K_3 \tag{39}$$

Following the same analysis in previous section, we building the Hamiltonian function

$$\begin{aligned} H = A_T &+ \lambda_1(R_QC - K_1) + \lambda_2(R^2CC_1 - K_2) \\ &+ \lambda_3(R - K_3R_1) \end{aligned} \tag{40}$$

Taking the partial derivatives of R, R_1, R_Q and C, we can get the expression of the area for resistors and capacitors respectively:

$$A_R = \frac{2R}{R_d} + \frac{R_1}{R_d} + \frac{R_Q}{R_d} = -2\lambda_2K_2 - \lambda_1K_1 \tag{41}$$

$$A_C = \frac{C}{C_d} + \frac{C_1}{C_d} = -2\lambda_2K_2 - \lambda_1K_1 \tag{42}$$

Comparing the equations (41) and (42), we can see that the area of resistors is the same as the area of capacitors when minimum passive area is obtained. This results is still independent of constrain K_1, K_2, K_3.

3.2 Tow-Thomas Biquad with the Transconductance T-network

After the transconductance network has been introduced into the circuit, all the large resistors are replaced by T-networks [2]. R, R1, Rq in figure 5 are all replaced by the T-networks composed by three resistors, R_1, R_2 and R_3 with the relationship: $R \cong \frac{R_1}{R_2} \cdot R_3 = KR_3$.There is similar transformation and all the analysis is the same as the first-order one presented in section 2. From that, we know that this conjecture is still hold in the second-order filters with T-network. Thus this strategy shows practical importance in the design of the RC-filters for audio frequency applications.

3.3 Second-order Bridge-T Feedback

Another popular second-order system is studied in this section.

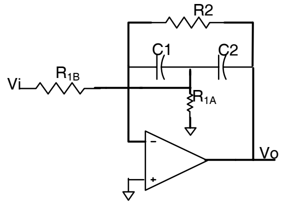

Figure 6 Second-order Bridge-T Feedback Circuit

The transfer function is shown below:

$$T(s) = \frac{\frac{1}{R_{1B}C_2} S}{S^2 + S\frac{(C_1 + C_2)}{R_2C_1C_2} + \frac{(R_{1A} + R_{1B})}{R_{1A}R_{1B}R_2C_1C_2}} \quad (43)$$

Three constrains are set:

$$\frac{C_1R_2}{R_{1B}(C_1 + C_2)} = K_1 \quad (44)$$

$$\frac{(R_{1A} + R_{1B})}{R_{1A}R_{1B}R_2C_1C_2} = K_2 \quad (45)$$

$$\frac{(C_1 + C_2)}{R_2C_1C_2} = K_3 \quad (46)$$

Total area and Hamiltonian function shown as below

$$A_T = \frac{R_2 + R_{1A} + R_{1B}}{R_d} + \frac{C_1 + C_2}{C_d} \quad (47)$$

$$H = A_T + \lambda_1[\frac{C_1R_2}{R_{1B}(C_1 + C_2)} - K_1] + \lambda_2[\frac{(R_{1A} + R_{1B})}{R_{1A}R_{1B}R_2C_1C_2} - K_2] + \lambda_3[\frac{(C_1 + C_2)}{R_2C_1C_2} - K_3] \quad (48)$$

Taking the partial derivatives of H respect to all the variables to achieve the following results.

$$A_R = \frac{R_{1A} + R_{1B} + R_2}{R_d} = 2\lambda_2K_2 + \lambda_3K_3 \quad (49)$$

$$A_C = 2\lambda_2K_2 + \lambda_3K_3 \quad (50)$$

Comparing the equation (49) and (50), it proves that the area for resistors equates the area of the capacitors when the optimum passive area is achieved.

4. OTHER ACTIVE RC FILTER STRUCTURES

The first-order and second-order filter structures studied in the previous two sections represent some of the more popular first-order and second-order active RC filters. Although they are of interest in their own right, they were also selected because they represent fundamentally different filter architectures with varying numbers of components, component spreads and internal nodes. Since the minimum total area for the resistors and capacitors was always achieved when the total resistor area equaled the total capacitor area, the question about whether this is a property inherent in all active RC filters naturally arises. Needles to say, when the authors investigated this problem, essentially all active filters that were considered, with the exception associated with the resistors needed to build finite gain amplifiers as described below, possessed this property. Thus, it is conjectured that this property is shared by a much larger number of useful active RC filters. The exception has to do with circuits that use resistors to build finite gain amplifiers. The Tow-Thomas biquad discussed above included such a finite gain amplifier as do some of the Sallen and Key filters. Beyond observing that such filters are characterized by a hinged graph of passive components in contrast to all examples considered in the previous sections in which the graph of passive components are not hinged, we will not attempt to conjecture what topological properties of the filter are necessary for the equal resistor/equal capacitor area property to hold.

5. CONCLUSION

The problem of minimizing area for passive components in audio frequency active filters has been addressed for the first time in closed-form. In this paper, several popular and useful first-order and second-order RC active filter structures with substantially different component ratios and with varying numbers of passive components were considered. These structures are all inherently continuous-time in nature and offer excellent linearity when compared to alternative continuous-time monolithic filter approaches based upon using transconductors or MOSFETs as resistance or transresistance elements. A detailed analysis of each of these structures showed that the total area for the resistors and capacitors is minimized when the total resistor area equals the total capacitor area irrespective of capacitance density, resistance density, pole locations or DC gain.

6. REFERENCES

[1] Anna M. Durham, William Reman-White and John B. Hughes, “High-Linearity Continuous-Time Filter in 5-V VLSI CMOS”, *IEEE J. Solid-State Circuits*, vol. 27. No.9, pp1270-1276, September 1992

[2] Rolf Schaumann, Mac E. Van Valkenburg “Design of Analog Filter” Oxford University Press 2001

[3] H. Fei and R. L. Geiger, "Low-Distortion Continuous-Time Integrated Filter for Low Frequency Applications", *Proc. of 2002 Midwest Symposium on Circuits and Systems*, Tulsa, OK, Aug 2002

[4] J. Harrison and N. Weste, “350 MHz opamp-RC filter in 0.18 um CMOS”, *Electronics Letters*, vol. 38, No.6, pp259-260, March 2002

[5] Anna M. Durham and William Redman-White “Integrated Continuous-Time Balanced Filters for 16-b DSP Interfaces”, *IEEE J. Solid-State Circuits*, vol. 28. No.7, pp835-839, July 1993